Chapter 9: Center of Mass and Momentum
Thursday February 19th

- Mini Exam III (25 minutes)
- Center of mass
- Newton's 2nd law for a system of particles
- Linear momentum and Newton's 2nd law
- Momentum conservation
- Impulse
- Example problems, *i*clicker and demos

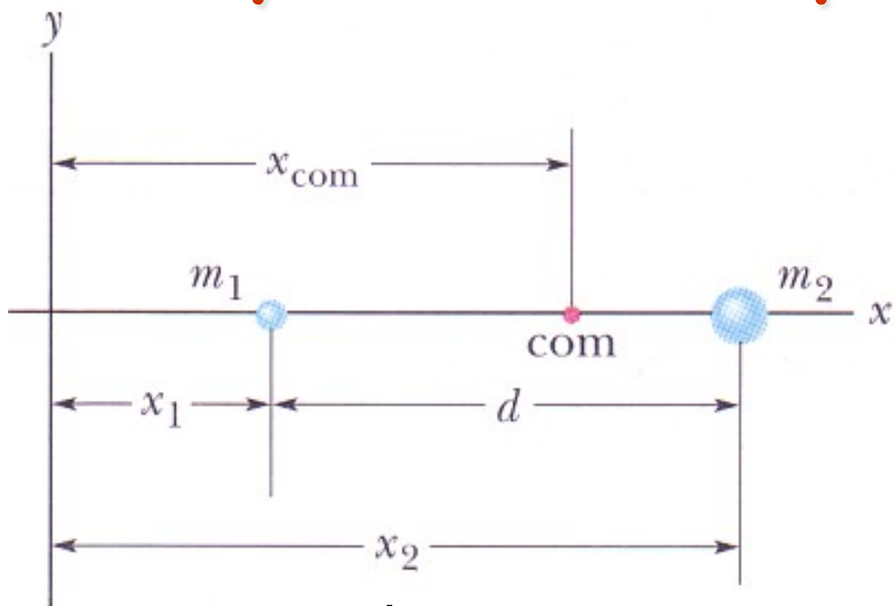
Reading: up to page 139 in Ch. 9 (skip Ch. 8 for now)

Chapter 9: Center of mass

- Much of physics involves looking for ways to simplify complicated interactions.
- An example is the motion of a baseball bat thrown into the air.
- If one looks carefully, there is a special point of the bat that moves in a simple parabolic path.
- That special point is the **center of mass** of the bat.

The center of mass of a body or a system of bodies is the point that moves as though all of the mass were concentrated there, and all external forces were applied there.

Center of mass for a system of two particles



• We define:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

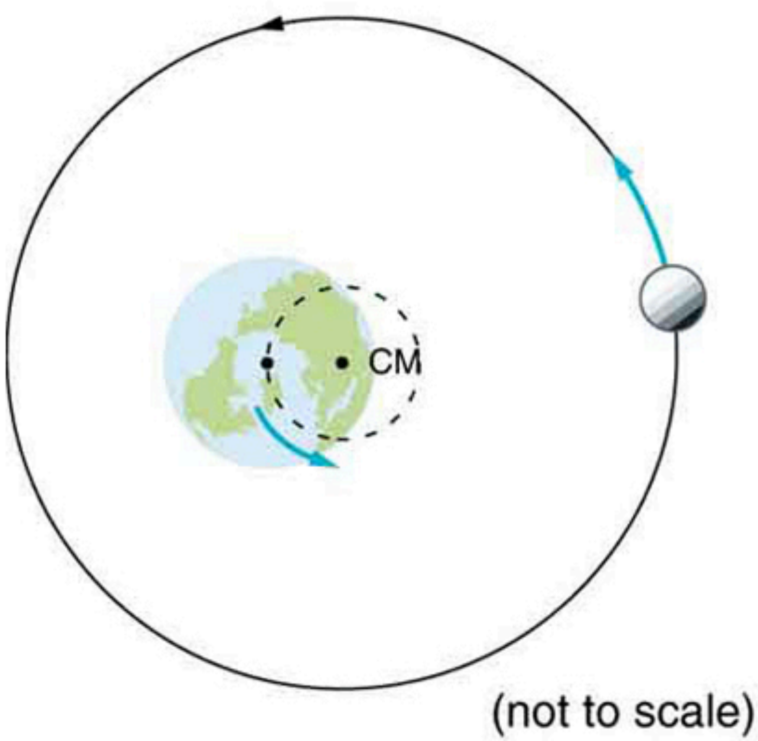
• In general:

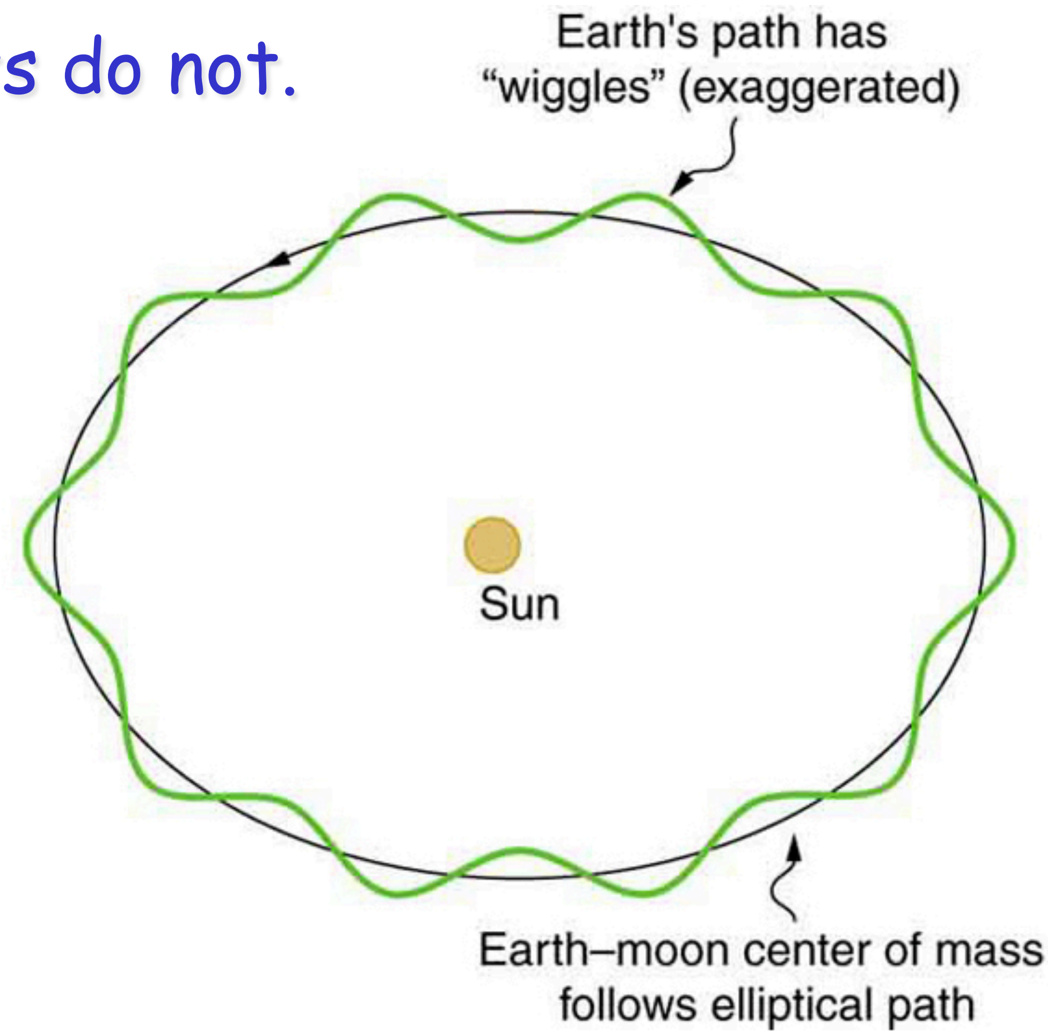
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{M}$$

where M is the total mass ($M = m_1 + m_2$) of the system

Newton's second law for a system of particles

- Although the center of mass is just a point, it moves like a particle whose mass is equal to the total mass of the system.
- The individual objects do not.





Extending to a system of *n* particles

$$x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots\cdots + m_nx_n}{M}$$

$$= \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad\qquad M = \sum_{i=1}^{n} m_i$$

- Here, i is a running number, or index, that takes on all integer values from 1 to n.
- In three-dimensions:

$$x_{\text{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i; \quad y_{\text{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i; \quad z_{\text{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

$$\vec{r}_{\text{cm}} = x_{\text{cm}}\hat{i} + y_{\text{cm}}\hat{j} + z_{\text{cm}}\hat{k}$$

Extending to a system of *n* particles

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- Here, i is a running number, or index, that takes on all integer values from 1 to n.

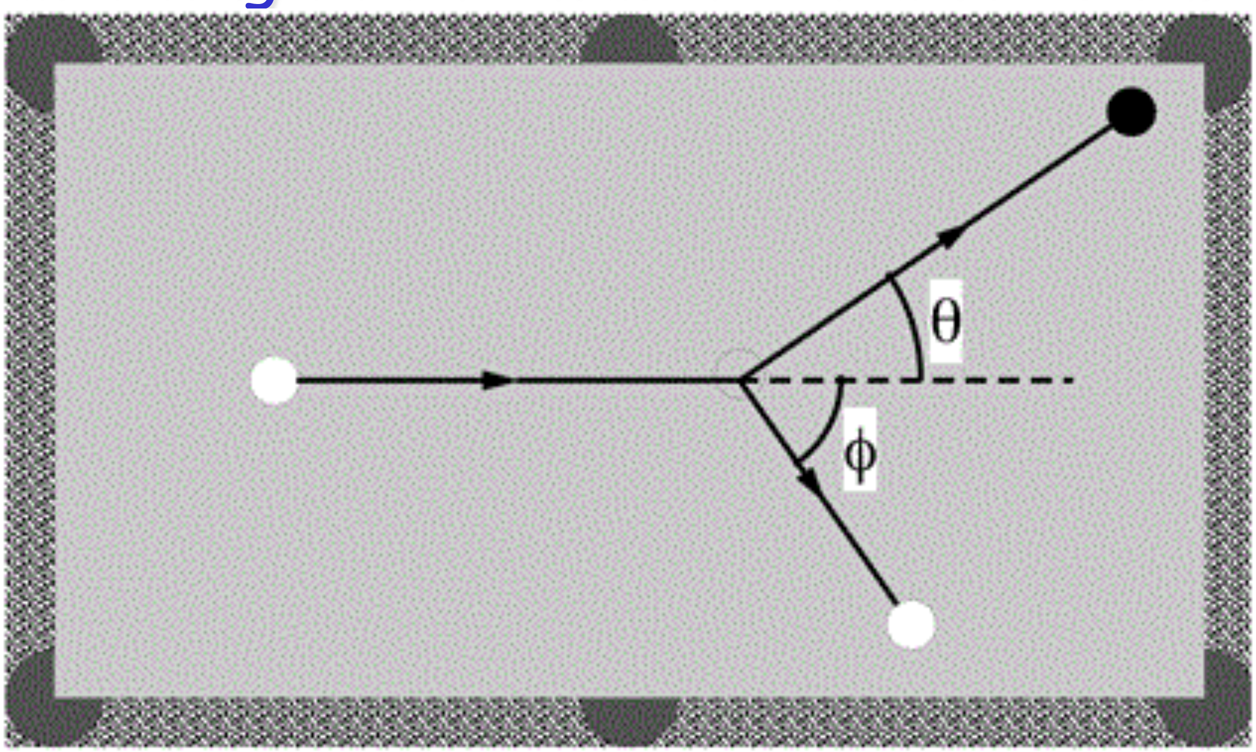
- For continuous distributions of matter:

$$\vec{r}_{\text{cm}} = \frac{1}{M}\int \vec{r}\, dm = \frac{1}{M}\int \rho(\vec{r})\vec{r}\, dV$$

Newton's second law for a system of particles

If no net (external) force acts upon a system of particles, then the motion of the center of mass of the system will remain unchanged.

- For example, in a collision between two billiard balls, the center of mass of the two balls continues to move as though there was never a collision.



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- For example, in a collision between two billiard balls, the center of mass of the two balls continues to move as though there was never a collision.

- This fact proves to be extremely useful when analyzing collisions (Tuesday's class).

- If a force does act on a system, then:

$$\vec{F}_{net} = M\,\vec{a}_{\mathrm{cm}} \qquad \text{(system of particles)}$$

Linear momentum and Newton's 2nd Law

- Definition of linear momentum, $\vec{p}$:

$$\vec{p} = m\vec{v}$$

- If one takes the derivative (constant mass):

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{net}$$

The time rate of change of momentum of a particle is equal to the net force acting on the particle and is in the direction of the force.

Linear momentum and Newton's 2nd Law

- Definition of linear momentum, $\vec{p}$:

$$\vec{p} = m\vec{v}$$

- Holds also if mass is changing (LONCAPA problem):

$$\frac{d\vec{p}}{dt} = \vec{v}\frac{dm}{dt} = \vec{F}_{net}$$

The time rate of change of momentum of a particle is equal to the net force acting on the particle and is in the direction of the force.

- If both mass and velocity are changing, it is a little more complicated, e.g., a rocket ejecting hot gas. We'll look at this next week.

Linear momentum of a system of particles

- A system of n particles has a total linear momentum given by:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots\cdots + \vec{p}_n$$

$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots\cdots + m_n\vec{v}_n$$

$$= M\,\vec{v}_{\mathrm{cm}}$$

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

$$\frac{d\vec{P}}{dt} = M\vec{a}_{\mathrm{cm}} = \vec{F}_{net}$$

Conservation of linear momentum

- For a system of *n* particles, if no net force acts on the system:

$$\vec{P} = \text{constant} \qquad \text{(closed, isolated system)}$$

If no net external force acts on a system of particles, the total linear momentum of the system cannot change

$$\begin{pmatrix} \text{total linear momentum} \\ \text{at some initial time } t_i \end{pmatrix} = \begin{pmatrix} \text{total linear momentum} \\ \text{at some later time } t_f \end{pmatrix}$$

- These are vector equations, *i.e.*

$$P_x = \text{constant}; \quad P_y = \text{constant}; \quad P_z = \text{constant}$$

If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.